Risk and return

The basics

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Outline

1. Stand alone measures
   - Definitions and notation
   - Expected return and risk
   - Bringing risk and return together
2. Portfolio measures
   - Portfolio return
   - Portfolio risk
3. Required return
   - Total vs. market risk
   - Calculating required return
4. Summary
   - What you saw in this presentation...

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Historical, expected, required

- Historical returns $\left(\overline{R}\right)$: The realized or after-the-fact return.
- Expected return $\left(E\left[R\right] = \widehat{R}\right)$: The return that equates the current price with expected future cash flows.
- Required return (R): The return required for delayed compensation and market risk.
- Notation:
  - Summation

$$\sum_{i=1}^{n} ☺_i = ☺_1 + ☺_2 + \cdots + ☺_n$$

  - Product-ation

$$\prod_{i=1}^{n} ☺_i = ☺_1 \times ☺_2 \times \cdots \times ☺_n$$

Arithmetic vs. geometric mean

- Arithmetic mean is a forward-looking measure. Statistically it is BLUE (best linear unbiased estimator). It is the best estimate of next period's value:

$$\text{arithmetic mean} = \widehat{R} = \frac{1}{n}\sum_{t=1}^{n} R_t = \frac{1}{n}(R_1 + R_2 + \cdots + R_n)$$

- Geometric mean is a backward-looking measure. it measures the actual returns (after-the-fact in IFM10 terminology) realized in the past:

$$\text{geometric mean} = \overline{R} = \left(\prod_{t=1}^{n} RR_t\right)^{1/n} - 1 = (RR_1 \times RR_2 \times \cdots \times RR_n)^{1/n} - 1$$

  - You *must* use relative returns RR_t when computing the geometric mean.
  - IFM10 uses $\overline{R}$ to represent the realized or after the fact return. In the past I have used $\overline{R}$ to represent the arithmetic mean and G for geometric mean. From this point forward I will use $\widehat{R}$ for arithmetic mean and $\overline{R}$ for geometric mean.

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Historical vs. probability input data

- When dealing with stock returns you are typically given historical data.
- When dealing with sales data you are typically given sales expectations for various potential states of the world.
- The calculations are the same whether you are dealing with returns, dollar amounts, heights, number of units, etc. Here I show return examples.

Historical data

Year (t)	R_t
1	10%
2	-5%
3	7%

Probabilities

Demand (i)	Pr_i	R_i
Weak	0.2	-5%
Normal	0.6	7%
Strong	0.2	10%

Calculations

Given historical data r_t

Expected return:

$$E[R] = \widehat{R} = \frac{1}{n}\sum_{t=1}^{n} R_t \tag{1}$$

The risk (variance) is calculated as:

$$\sigma^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left(R_t - \widehat{R}\right)^2 \tag{2}$$

Given probabilities Pr_i and returns R_i

Expected return:

$$E[R] = \widehat{R} = \sum_{i=1}^{n}(R_i \times Pr_i) \tag{3}$$

The risk (variance) is calculated as:

$$\sigma^2 = \sum_{i=1}^{n}\left(\left(R_i - \widehat{R}\right)^2 \times Pr_i\right) \tag{4}$$

- See if you can take the data from the previous slide and compute expected returns and variances.
- Note the standard deviation σ is the square root of the variance σ^2: $\sigma = \sqrt{\sigma^2}$

One more way to calculate expected return

- Recall the definition of expected return: The return that equates the current price with expected future cash flows.
- Expressed mathematically:

$$P_0 = \sum_{t=1}^{n} \frac{D_t}{(1+R)^t} \tag{5}$$

- If we impose some structure on Eq. (5), specifically constant growth of dividends at rate g, the new equation is:

$$P = \frac{D_1}{R-g} \tag{6}$$

  where $D_1 = D_0\,(1+g)$
- We can rearrange Eq. (6) to arrive at another expected return measure:

$$\widehat{R} = \frac{D_1}{P_0} + g = \frac{D_0}{P_0}\,(1+g) + g \tag{7}$$

  In other words, given the current dividend D_0, the current stock price P_0, and the expected dividend growth rate g, we can compute the expected return $\widehat{R}$.

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Coefficient of variation

- Someone tells you they “beat the market” with an average (arithmetic) return of 15% over the past 5 years.
- Did they beat the market? The answer is you do not know.
- You need to know what level of risk was involved.
- Insert coefficient of variation:

$$CV = \frac{\sigma}{\widehat{R}}$$

- CV measures units of total risk per unit of expected return.
- Lower CV means less risk and more return $\rightarrow$ lower is better!
- Your friend “beat the market” if they had a lower CV than the market. ☺

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Portfolio expected return $E\left[R_p\right] = \widehat{R}_p$

- You can compute the portfolio expected return two ways:
  1. Compute portfolio return by time period t or state of the world i. Treat $R_{p,t}$ or $R_{p,i}$ as an individual security. Then apply formulas (1) and (3) to $R_{p,t}$ and $R_{p,i}$, respectively.
  2. For each security j compute $E\left[R_j\right] = \widehat{R}_j$ then apply this formula:

$$E\left[R_p\right] = \widehat{R}_p = \sum_{j=1}^{n} \left(w_j \widehat{R}_j\right) = w_1 \widehat{R}_1 + w_2 \widehat{R}_2 + \cdots + w_n \widehat{R}_n \qquad (8)$$

- Presuming 40% in stock A ($w_a = 0.40$) and 60% in stock B ($w_b = 0.60$):

Historical data

Year (t)	$R_{a,t}$	$R_{b,t}$	$R_{p,t}$
1	10%	3%	5.8%
2	-5%	4%	0.4%
3	7%	6%	6.4%

Probabilities

Demand (i)	Pr_i	$R_{a,i}$	$R_{b,i}$	$R_{p,i}$
Weak	0.2	-5%	3%	-0.2%
Normal	0.6	7%	4%	5.2%
Strong	0.2	10%	6%	7.6%

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Portfolio risk

- Unlike portfolio expected return, portfolio risk is not the weighted average of the individual variances: $\sigma_p^2 \neq \sum w_i \sigma_i^2$.
- In reality, the portfolio risk will be less. A brief look at portfolio theory is provided in later slides.
- For now, we have enough data and tools to compute portfolio risk without worrying about portfolio theory.
- Begin with portfolio return by time period $R_{p,t}$ or state of the world $R_{p,i}$ and the portfolio expected return $\widehat{R}_p$.

Given historical data R_t

$$\sigma_p^2 = \frac{1}{n-1} \sum_{t=1}^{n} \left(R_{p,t} - \widehat{R}_p\right)^2 \tag{9}$$

Given probabilities Pr_i and returns R_i

$$\sigma_p^2 = \sum_{i=1}^{n} \left(\left(R_{p,i} - \widehat{R}_p\right)^2 \times Pr_i \right) \tag{10}$$

Portfolio theory: co-movement

- The *absolute* measure of co-movement is covariance σ_{ab}.

Given historical data R_t

$$\sigma_{ab} = \frac{1}{n-1}\sum_{t=1}^{n}\left(R_{a,t} - \widehat{R}_a\right)\left(R_{b,t} - \widehat{R}_b\right) \quad (11)$$

Given probabilities Pr_i and returns R_{ai} and R_{bi}

$$\sigma_{ab} = \sum_{i=1}^{n}\left(R_{a,i} - \widehat{R}_a\right)\left(R_{b,i} - \widehat{R}_b\right) Pr_i \quad (12)$$

- The *relative* relationship between co-movements of returns of returns is the correlation coefficient ρ:

$$\rho = \begin{cases} +1.0 & \text{perfect positive correlation} \\ 0 & \text{no correlation} \\ -1.0 & \text{perfect negative correlation} \end{cases}$$

- Covariance and correlation are related in the following manner:

$$\rho_{ab} = \frac{\sigma_{ab}}{\sigma_a \sigma_b} \quad (13)$$

Portfolio theory: portfolio risk

- For the two-security case portfolio risk σ_p is calculated as:

$$\sigma_p^2 = w_a^2\sigma_a^2 + w_b^2\sigma_b^2 + 2w_a w_b \rho_{ab}\sigma_a\sigma_b \tag{14}$$

- In general for n securities:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\neq j}}^{n} w_i w_j \sigma_{ij} \tag{15}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{16}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij}\sigma_i\sigma_j \tag{17}$$

- Note: as the number of securities increases, the importance of each security's variance decreases.
- For fun, compute the portfolio risk using Eqs. (9) and (14).

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Total and market risk

- The total risk measure σ has two components
  - Diversifiable or idiosyncratic risk that we do not bother measuring.
  - Non-diversifiable or market risk measured by β.
- An investor is not rewarded for bearing diversifiable risk. Why should you receive additional return per unit of risk that you could have eliminated?
- You are compensated for bearing non-diversifiable market risk.
- For more information see IFM10 page 52.

Measuring market risk

- Market risk is measured by β.
- Beta is obtained by performing the market model regression:

$$R_t = \alpha + \beta R_{m,t}$$

- Where R_t is the return of the stock in question at time t and $R_{m,t}$ is the return of the market at time t.

β	**Interpretation**
$\beta < 1$	Below average risk
$\beta = 1$	Average risk
$\beta > 1$	Above average risk

- Calculation of beta in a portfolio is straightforward:

$$\beta_p = \sum_{j=1}^{n} w_j \beta_j$$

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From market risk to required return

- When you purchase a stock you are (1) delaying your consumption and (2) taking on *market* risk. You must be compensated for both.
- The total compensation is called the required return.
- Required return R is measured by the Security Market Line (SML) or CAPM equation:

$$R = R_f + \beta\left(E\left[R_m\right] - R_f\right)$$

where

$$\begin{aligned}
R_f &= \text{compensation for delayed consumption} \\
\beta &= \text{number of units of market risk} \\
E\left[R_m\right] - R_f &= \text{market risk premium} \\
&= \text{compensation for one unit of market risk} \\
\beta\left(E\left[R_m\right] - R_f\right) &= \text{compensation for bearing market risk}
\end{aligned}$$

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Many things...

- The distinction between historical return $\left(\overline{R}\right)$, expected return $\left(\widehat{R}\right)$, and required return (R).
- How to calculate historical return with the geometric mean: $\overline{R}$
- How to calculate expected return given historical data (arithmetic mean), when given a probability table, and with current market information $\left({}^{D_1}/_{P_0},\ g\right)$: $\widehat{R}$
- How to calculate risk (variance): variance σ^2 for individual securities; covariance σ_{ab} and ρ_{ab} between securities.
- How to calculate portfolio expected return $\widehat{R}_p$ and portfolio risk σ_p^2 two ways.
- How to calculate required return using CAPM: $R_i = R_f + \beta\left(E\left[R_m\right] - R_f\right)$
    - Note: this applies to individual assets and portfolios.